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Paper-1

phase velocity and Group velocity

A wave motion is a kind of disturbance which travels in the medium due to vibrational motion of the particles of the medium about their mean position, motion is transferred from one particle to another. particles of the medium has simple harmonic motion about their mean position and do not move with the wave. Every particles start vibrating after same time from its precedent particle and the phase changes from one particle to another. Actually the observed wave is a phase relation of these particles and not the progressive motion of the particles in the medium.

phase velocity or wave velocity :- when a monochromatic wave travels in a medium, then the velocity of propagation of the wave in the medium is know as wave velocity. For example, the equation of the motion of a plane harmonic wave in +ve x-direction is

$$y = y_m \sin(\omega t - kx)$$

where y_m is the amplitude, ($\omega = 2\pi n$) is angular frequency and K is propagation constant of the wave. By definition, the ratio of angular velocity ω and propagation constant K is known as wave velocity v. Thus

$$v = \frac{\omega}{K}$$

$(\omega t - Kx)$ is the phase of wave motion. Therefore, plane or wave fronts of constant phase are define by

$$\omega t - Kx = \text{Constant}$$

Differentiating it w.r.t. t we get

$$\omega - K\frac{dx}{dt} = 0$$

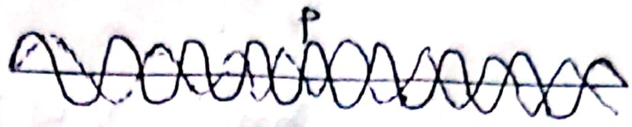
$$\text{or,}\ \frac{dx}{dt} = \frac{\omega}{K}$$

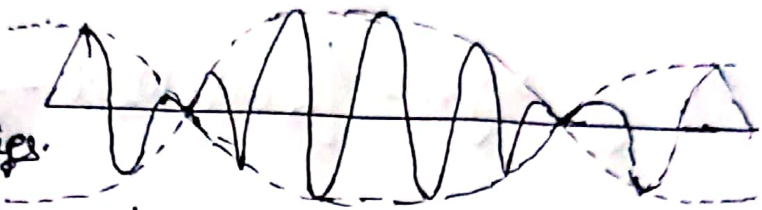
This is the wave velocity v. Thus the wave velocity is the velocity with which the plane of constant phase propagates in the medium. For this reason. wave velocity is also known as phase-velocity.

Group velocity :- In particle, we obtain ~~ptt~~ pulses, and not the monochromatic wave. A pulse consists of more than one wave having the frequencies slightly different from each other. By superposition of these waves, a wave packet or wave-group is obtained. When such a group travels a medium, then phase velocity of different components of this group are different. But we observed the velocity with which the maximum amplitude of the group travels the medium. This is known as 'Group velocity'.

Thus, group velocity is the velocity with which the energy of group is transmitted. waves of group travels within the group whith their phase-velocity.

Figure shows two waves of equal amplitudes having slightly difference frequencies.





The waves are propagating from left to right. At an instant, the two waves are in phase at P so that the maximum aplitude of the group of these waves is also at P. After some time, the position of maximum amplitude, relative to waves travels towards left. Thus the group velocity is always less than wave velocity.

Expression for Group velocity! – Let a group-wave consists of two waves of equal amplitude y_m having angular frequencies ω_1 and ω_2 differing slightly. The propagation constant are K_1 and K_2. The displacement due to the wave are

$$y_1 = y_m \sin(\omega_1 t - K_1 x) \quad \text{——— (1)}$$

$$y_2 = y_m \sin(\omega_2 t - K_2 x) \quad \text{——— (2)}$$

The resultant displacement due to Superposition is

$$y = y_1 + y_2$$

$$= y_m [\sin(\omega_1 t - K_1 x) + \sin(\omega_2 t - K_2 x)]$$

$$= 2y_m \sin\left[\frac{1}{2}(\omega_1+\omega_2)t - \frac{1}{2}(K_1+K_2)x\right] \cdot \cos\left[\frac{1}{2}(\omega_1-\omega_2)t - \frac{1}{2}(K_1-K_2)x\right]$$

or, $$y = 2y_m \cos\left[\frac{1}{2}(\omega_1-\omega_2)t - \frac{1}{2}(K_1-K_2)x\right] \cdot \sin\left[\frac{1}{2}(\omega_1+\omega_2)t - \frac{1}{2}(K_1+K_2)x\right]$$

This expression represent a wave group having a frequency $\frac{1}{2}(\omega_1+\omega_2)$ and amplitude $2y_m \cos\left[\frac{1}{2}(\omega_1-\omega_2)t - \frac{1}{2}(K_1-K_2)x\right]$ Thus the amplitude of wave-group is modulated in time and distance both. The maximum value of amplitude is $2y_m$. This envelope is shown by dotted Curve in figure. The velocity with which this envelope propagates that is velocity of maximum amplitude of the Group is

$$V_0 = \frac{\omega_1 - \omega_2}{K_1 - K_2} = \frac{\Delta\omega}{\Delta K}$$

If the group Consists of many Components of different frequencies having an infinitesimal frequency interval, then the expression for group velocity is

$$V_0 = \frac{d\omega}{dK}$$

Relation between Group velocity and phase velocity :— As $\omega = Kv$, where v is the wave velocity, therefore group velocity will be

$$V_0 = \frac{d\omega}{dk}$$

$$= \frac{d}{dk}(kv)$$

$$= v + K\frac{dv}{dk}$$

But

$K = 2\pi/\lambda$, where λ is the wavelength.

$$\therefore V_0 = v + \frac{2\pi}{\lambda} \cdot \frac{dv}{d(2\pi/\lambda)}$$

$$= v + \frac{1}{\lambda} \cdot \frac{dv}{d(1/\lambda)}$$

But $d(1/\lambda) = -(1/\lambda^2)$ therefore

$$\boxed{V_0 = v - \lambda\frac{dv}{d\lambda}}$$

This is the relation between group velocity (V_0) and wave velocity in a dispresive medium.

If the medium is non-dispersive, the $v = \frac{\omega}{K}$ = Constant, so that $\frac{dv}{d\lambda} = 0$. Thus

$$V_0 = V$$

If free space, the group velocity of the light waves is the Same as wave velocity.

Normally $dv/d\lambda$ is positive. Therefore group velocity is Smaller than wave-velocity. But for anomalous dispersion, the value of V_0 is greater than v.